Monomial Mappings and Hilbert Modular Surfaces

Yasemin Kara joint work with John H.Hubbard

Let A be a matrix in $\mathrm{SL}_2(\mathbb{Z})$ with $|\operatorname{Tr} A| > 2$. Denote by $\lambda^{\pm}$ the two eigenvalues, with $|\lambda^{+}| > 1$ and $|\lambda^{-}| < 1$. It is convenient to suppose further that $\lambda^{\pm} > 0$.

The monomial map $M_A : (\mathbb{C}^*)^2 \to (\mathbb{C}^*)^2$ is defined by

$$M_A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^a y^b \\ x^c y^d \end{pmatrix}.$$

The map $M_A : (\mathbb{C}^*)^2 \to (\mathbb{C}^*)^2$ is an isomorphism, but of course is not an isomorphism from $\mathbb{P}^2 \to \mathbb{P}^2$: the three points $[0:0:1], [0:1:0], [1:0:0]$ are points of indeterminacy for either M_A or $M_{A^{-1}}$. We will make an infinite number of blow-ups in $\mathbb{P}^2$ to make a compact space X_A in which $(\mathbb{C}^*)^2$ is dense, and such that M_A extends to an "isomorphism" $\overline{M}_A : X_A \to X_A$.

Our interest in monomial mappings largely springs from trying to understand the resolution of singularities at the cusps of Hilbert modular surfaces $S_K = (\mathbb{H} \times \mathbb{H})/PSL_2(O_K)$ where K is a real quadratic field and O_K is its ring of integers.