

On downward half Cauchy sequences of real numbers

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In this paper, we introduce and investigate the concepts of down continuity and down compactness. A real valued function f on a subset E of \mathbf{R} , the set of real numbers is down continuous if it preserves downward half Cauchy sequences, i.e. the sequence $(f(\alpha_n))$ is downward half Cauchy whenever (α_n) is a downward half Cauchy sequence of points in E , where a sequence (α_k) of points in \mathbf{R} is called downward half Cauchy if for every $\varepsilon > 0$ there exists an $n_0 \in \mathbf{N}$ such that $\alpha_m - \alpha_n < \varepsilon$ for $m \geq n \geq n_0$. It turns out that the set of down continuous functions is a proper subset of the set of continuous functions.

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