Isometries of length 1 in purely loxodromic free Kleinian groups and trace inequalities

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Abstract

Let $\Xi = \{\xi_1, \xi_2, \ldots, \xi_n\}$ be a set of non-commuting isometries of the hyperbolic 3-space $\mathbb{H}^3$ so that $\Gamma = \langle \xi_1, \ldots, \xi_n \rangle$ for $n > 2$ is a purely loxodromic free Kleinian group. Let $\Phi_1^n = \Xi \cup \Xi^{-1}$ for $\Xi^{-1} = \{\xi_1^{-1}, \xi_2^{-1}, \ldots, \xi_n^{-1}\}$ and $d_\gamma z$ be the distance between the points z and $\gamma \cdot z$ in $\mathbb{H}^3$. Let $\Gamma_\varnothing^n$ be the set of all conjugates in Γ of length less than or equal to 3. In my talk I will prove that the trace inequality

$$|\mathrm{trace}^2(\gamma) - 4| + |\mathrm{trace}(\gamma\beta\gamma^{-1}\beta^{-1}) - 2| \geq 2\sinh^2\left(\tfrac{1}{4}\log\alpha_{1,n}\right)$$

holds provided that for some non-commuting isometries $\gamma, \beta \in \Phi_1^n$ the inequalities $d_\psi z_2 < \frac{1}{2}\log\alpha_{1,n}$ and $d_{\beta\gamma\beta^{-1}} z_2 \leq d_{\beta\gamma\beta^{-1}} z_1$ are satisfied for every $\psi \in \Gamma_\varnothing^n - \{\gamma, \gamma^{-1}, \beta^{-1}\gamma\beta, \beta^{-1}\gamma^{-1}\beta, \beta\gamma\beta^{-1}, \beta\gamma^{-1}\beta^{-1}\}$ for z_1 and z_2 the mid-points of the shortest geodesic segments connecting the axes of γ to $\beta\gamma\beta^{-1}$ and γ to $\beta^{-1}\gamma\beta$, respectively. Above $\alpha_{1,n}$ is the only real root greater than $(2n-1)^2$ of a certain fourth degree polynomial $p_n(x)$.

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