Jørgensen's Inequality and Purely Loxodromic 2–Generator Free Kleinian Groups

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Let ξ and η be two non–commuting isometries of the hyperbolic 3–space $\mathbb{H}^3$ so that $\Gamma = \langle \xi, \eta \rangle$ is a purely loxodromic free Kleinian group. For $\gamma \in \Gamma$ and $z \in \mathbb{H}^3$, let $d_\gamma z$ denote the distance between z and $\gamma \cdot z$. Let z_1 and z_2 be the mid-points of the shortest geodesic segments connecting the axes of ξ, $\eta\xi\eta^{-1}$ and $\eta^{-1}\xi\eta$, respectively. In my talk I will prove that if $d_\gamma z_2 < 1.6068...$ for every $\gamma \in \{\eta, \xi^{-1}\eta\xi, \xi\eta\xi^{-1}\}$ and $d_{\eta\xi\eta^{-1}} z_2 \leq d_{\eta\xi\eta^{-1}} z_1$, then $|\mathrm{trace}^2(\xi) - 4| + |\mathrm{trace}(\xi\eta\xi^{-1}\eta^{-1}) - 2| \geq 2\sinh^2\left(\frac{1}{4}\log\alpha\right) = 1.5937....$ Above $\alpha = 24.8692...$ is the unique real root of the quartic polynomial $21x^4 - 496x^3 - 654x^2 + 24x + 81$ that is greater than 9. Also generalisations of this inequality for finitely generated purely loxodromic free Klenian groups are conjectured.

MSC 2000: 54C30, 20E05, 26B25, 26B35
Keywords: Hyperbolic 3-space, Loxodromic Isometries, Hyperbolic Displacements, The Log 3 Theorem, Jørgensen's Inequality.

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